

Problem 3

Use equation (1.8) to find the fractions that are equivalent to the following repeating decimals:

$$0.55555\ldots$$

Solution

$$\begin{aligned}0.55555\ldots &= 0.5 + 0.05 + 0.005 + 0.0005 + \cdots \\&= \frac{5}{10} + \frac{5}{100} + \frac{5}{1000} + \frac{5}{10000} + \cdots \\&= \sum_{i=1}^{\infty} \frac{5}{10^i} \\&= 5 \sum_{i=1}^{\infty} \frac{1}{10^i} \\&= 5 \sum_{i=1}^{\infty} \left(\frac{1}{10}\right)^i \\&= 5 \left[-1 + \sum_{i=0}^{\infty} \left(\frac{1}{10}\right)^i \right] \\&= 5 \left[-1 + \frac{1}{1 - \left(\frac{1}{10}\right)} \right] \\&= 5 \left(-1 + \frac{10}{9} \right) \\&= 5 \left(\frac{1}{9} \right) \\&= \frac{5}{9}\end{aligned}$$